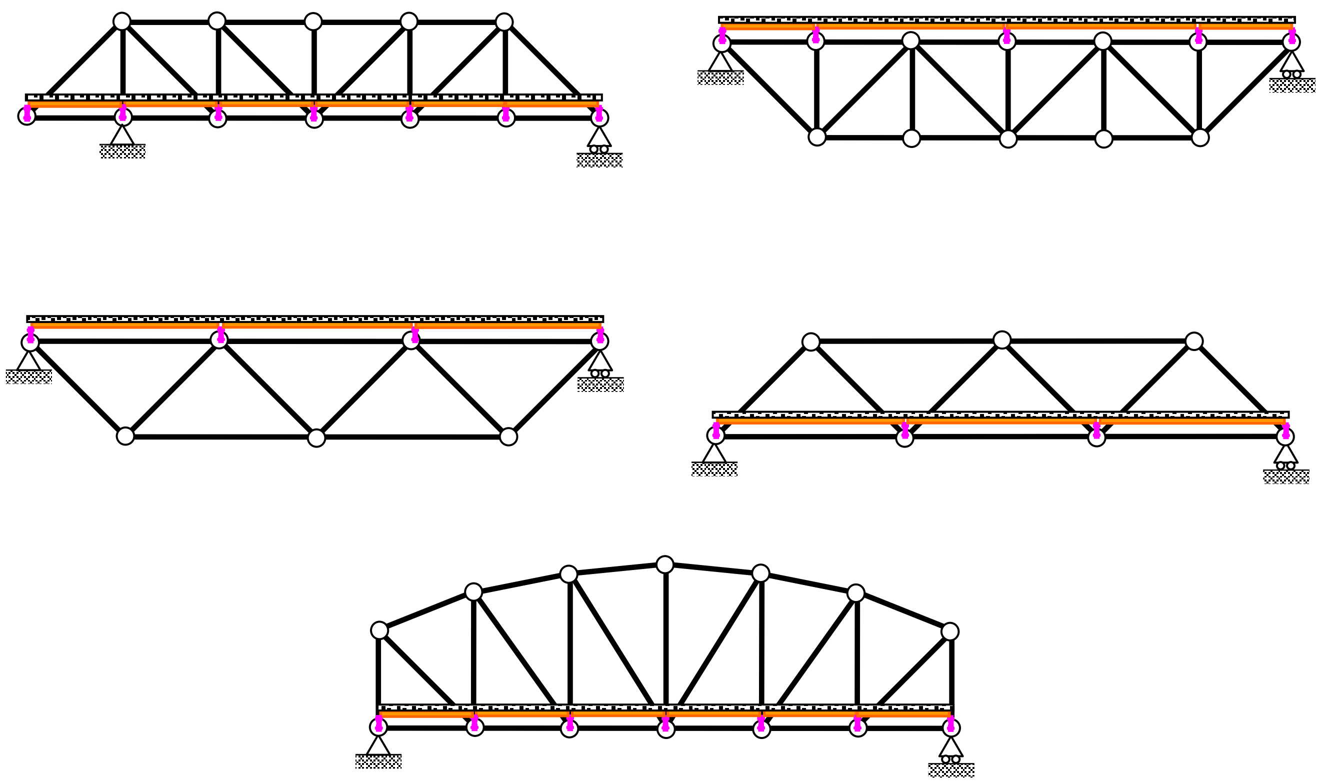
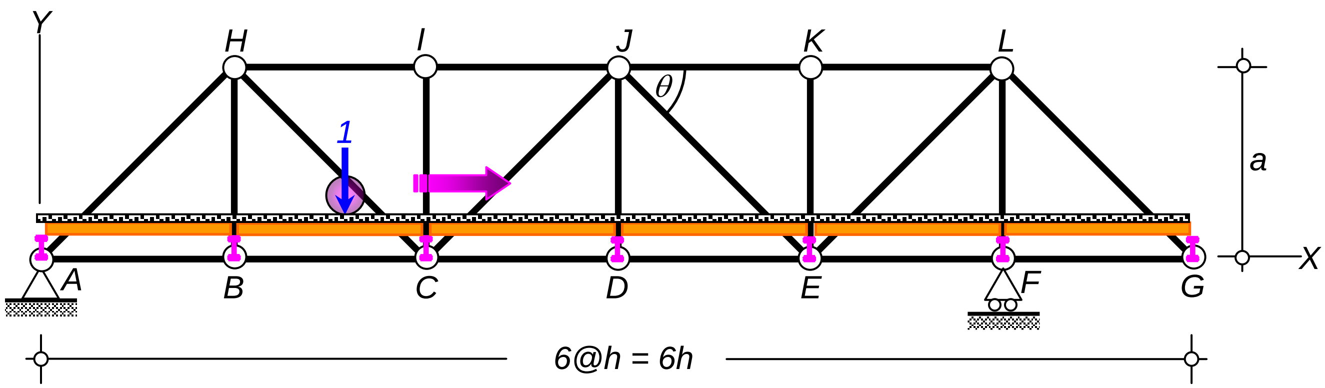
Remarks

- *The direct method can be used to construct the influence lines for statically determinate truss with any general configurations provided that the loading path is provided.*
- *A number of analyses must be performed in order to obtain values of the influence line at all panel points. This renders the method inefficient when the truss under consideration consists of several loading panels.*

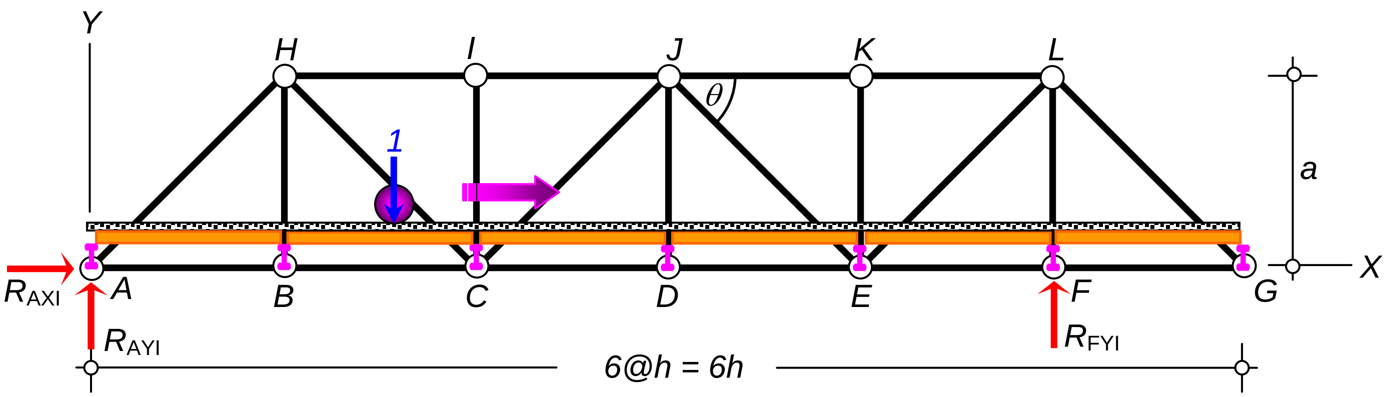
Construction of Influence lines by a truss-floor system analogy



For certain statically determinate trusses with a simple configuration and a loading path such as those shown in the above figures, the influence lines of the support reactions and the internal member forces can be alternatively obtained by using a method called ***truss-floor system analogy****. In this method, the truss under consideration is represented by a floor system of equivalent loading path, and the influence lines of certain quantities of this analogous structure is used as a basis for constructing the influence lines of the support reactions and the internal member forces for the truss. This analogy follows directly from the equivalence of the form of static equilibrium equations of the two systems.*



To demonstrate and clarify the procedure, let us consider the following statically determinate truss indicated above and construct the influence lines of the support reactions R_{AXI}, R_{AYI} and R_{FYI} and the internal forces of generic members F_{CDI}, F_{CJI}, F_{DJI}, and F_{IJI}.

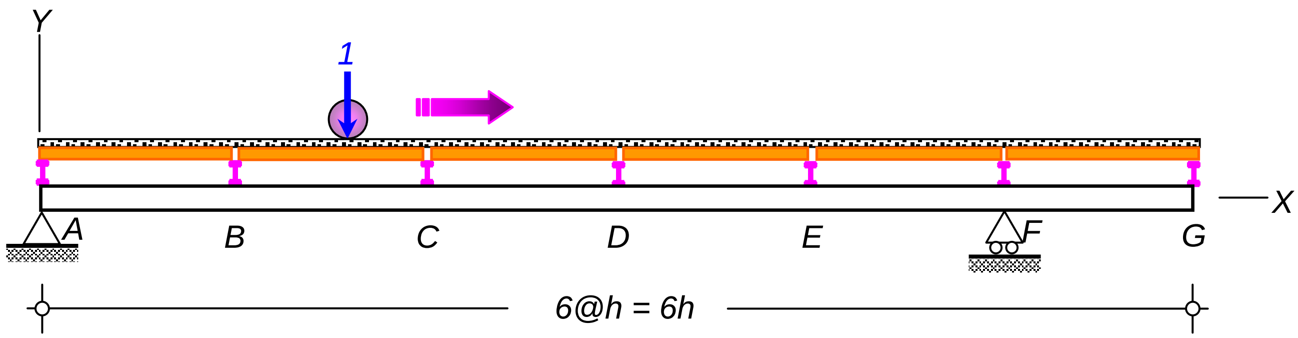


Free body diagram

From the free body diagram, the support reactions R_{AXI}, R_{AYI} and R_{FYI} of the truss can be computed from the following three equations of static equilibrium:

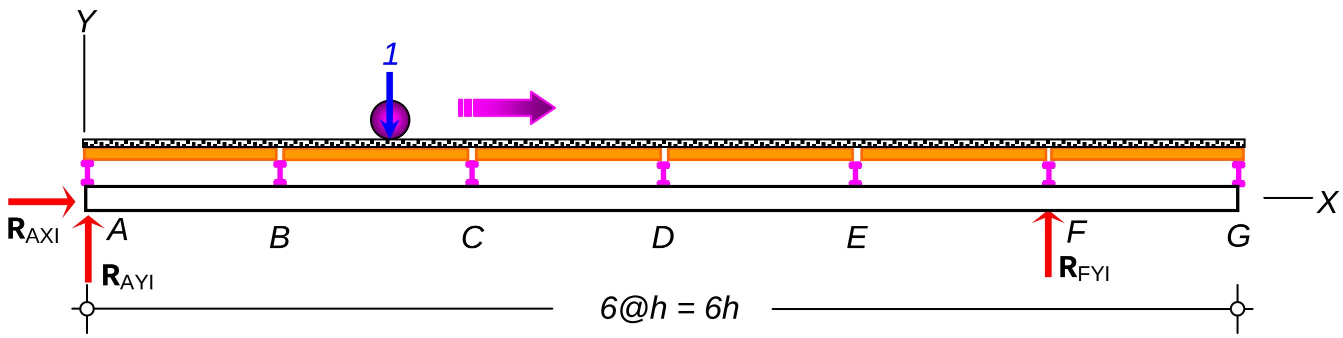
$$
\begin{aligned}
\Sigma F_X = 0 \quad &\Rightarrow \quad R_{AXI} = 0 \\
\Sigma F_Y = 0 \quad &\Rightarrow \quad R_{AYI} + R_{FYI} = 1 \\
\Sigma M_A = 0 \quad &\Rightarrow \quad R_{FYI}(5h) - 1 \cdot X = 0
\end{aligned}
\qquad (11)
$$

To demonstrate the analogy, we first form the equivalent floor system, i.e. a system with ***the same span****,* ***the same loading panels****, and* ***the same pattern of boundary conditions****. For above particular truss, the equivalent floor system is shown below.*



Equivalent floor system

The free body diagram of the floor system is given below. Note that script letters are used here to distinguish from those used for the influence lines of the truss.



Free body diagram

From the free body diagram, the support reactions $\mathbf{R}_{AXI}$, $\mathbf{R}_{AYI}$ *and* $\mathbf{R}_{FYI}$ *of the equivalent floor system can be computed from the following three equations of static equilibrium:*

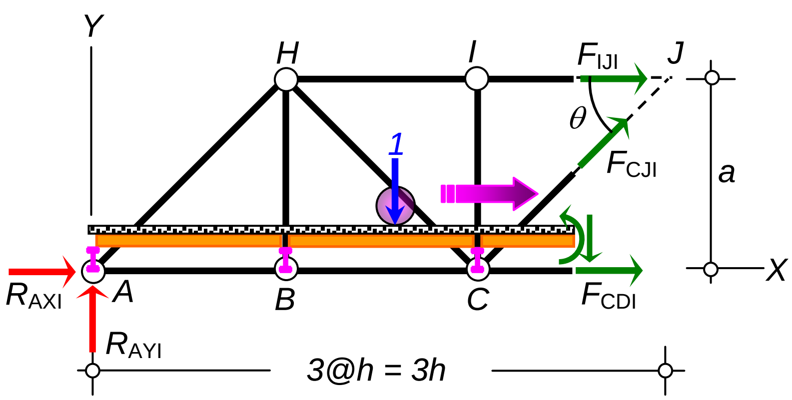
$$\begin{aligned} \Sigma F_X = 0 \quad &\Rightarrow \quad \mathbf{R}_{AXI} = 0 \\ \Sigma F_Y = 0 \quad &\Rightarrow \quad \mathbf{R}_{AYI} + \mathbf{R}_{FYI} = 1 \\ \Sigma M_A = 0 \quad &\Rightarrow \quad \mathbf{R}_{FYI}(5h) - 1 \cdot X = 0 \end{aligned} \qquad (12)$$

It is obvious from the two systems of equations (11) and (12) that three equations of static equilibrium for the truss and the equivalent floor system possess the same form for any locations of a moving unit load along the loading path. This implies that the influence lines of the support reactions of the two structures are identical, i.e.

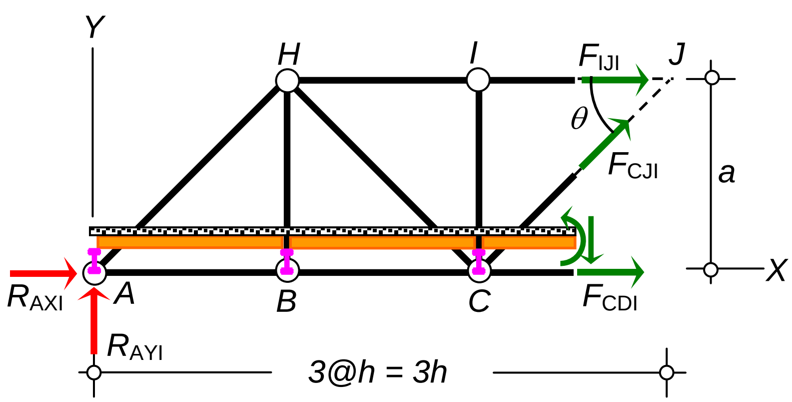
$$\begin{aligned} R_{AXI} &\equiv \mathbf{R}_{AXI} \\ R_{AYI} &\equiv \mathbf{R}_{AYI} \\ R_{FYI} &\equiv \mathbf{R}_{FYI} \end{aligned} \qquad (13)$$

The analogy (13) is useful for constructing the influence lines for the support reactions of truss structures since it is simply reduced to construct the influence lines of the support reactions of the equivalent floor system. Note that the influence lines of the support reactions for the floor system can be readily obtained using Müller-Breslau principle as previously discussed.

Now, we turn attention to form an analogy for the influence lines of the internal member forces. To determine the internal forces of the members CD, CJ and IJ, a method of sections is utilized with a fictitious section passing through those three members. The free body diagram of a part of the truss is shown below.



Unit load is on the left of the section



Unit load is on the right of the section

Free body diagram

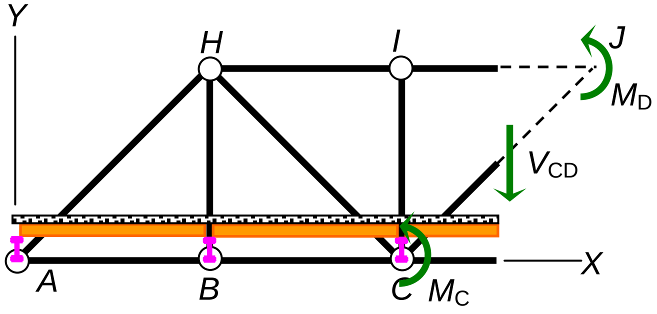
*Now, from the internal member forces appearing at the fictitious section, let form the resultant of these internal forces in terms of **the resultant internal shear force of the panel CD**, denoted by V_{CD}, and **the resultant internal moment about selected point C and J**, denoted by M_C and M_J, respectively:*

$$
\begin{aligned}
V_{CD} &\equiv -F_{CJI}\sin\theta \\
M_C &\equiv -F_{IJI}\cdot a \\
M_J &\equiv F_{CDI}\cdot a
\end{aligned}
\qquad (14)
$$

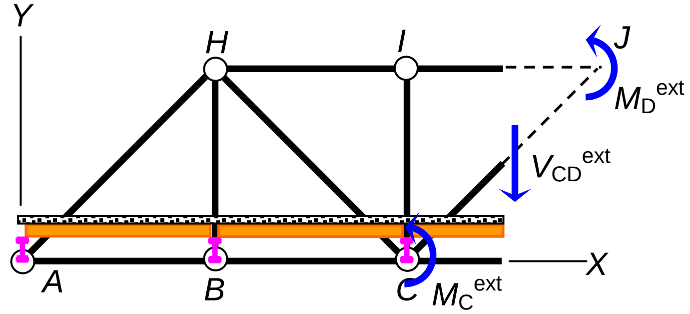
Similarly, the resultant external shear force of the panel CD and the resultant external moment at the same selected points due to the support reactions and loads transmitted to all panel points appearing in the FBD (through load transferring mechanism) can be computed as follow

$$
\begin{aligned}
V_{CD}^{ext} &\equiv R_{AYI} - L_{left} \\
M_C^{ext} &\equiv R_{AYI}(2h) - M_{C,left} \\
M_J^{ext} &\equiv R_{AYI}(3h) - M_{J,left}
\end{aligned}
\qquad (15)
$$

where L_{left} is the sum of load transmitted to all panel points appearing in the FBD; $M_{C,left}$ and $M_{J,left}$ are the sums of moment due to load transmitted to all panel points appearing in the FBD about points C and J, respectively. Note that all internal forces and external applied loads including the support reactions can be equivalently represented by the internal and external resultant shear forces and moments as shown in the figure below; the standard sign convention for shear and moment are used.



Resultant internal shear force and moments

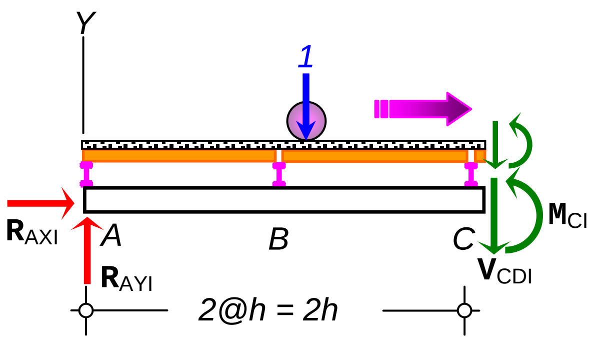


Resultant external shear force and moment

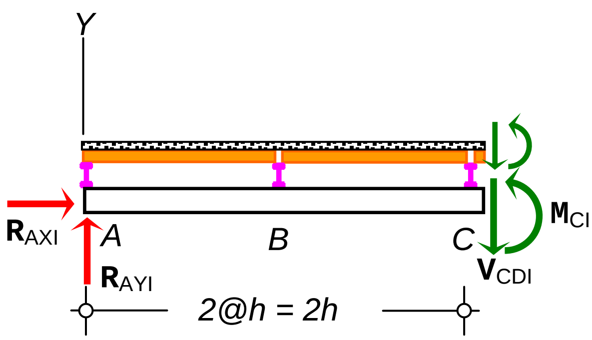
Since the structure is in equilibrium, any part of the structure must also be in equilibrium and this leads to

$$V_{CD} = V_{CD}^{ext}$$
$$M_C = M_C^{ext}$$
$$M_J = M_J^{ext} \quad (16)$$

Now let consider the equivalent floor system. By introducing a fictitious section within the panel CD at point just to the right of point C, the corresponding free body diagram is obtained below.



Unit load is on the left of the section



Unit load is on the right of the section

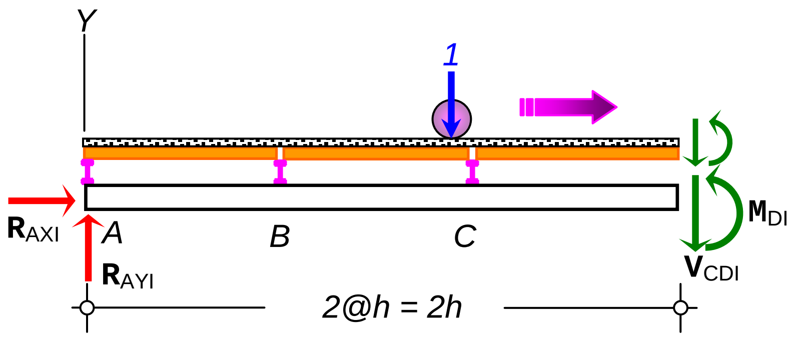
Free body diagram

Equilibrium of this portion of the floor system dictates that

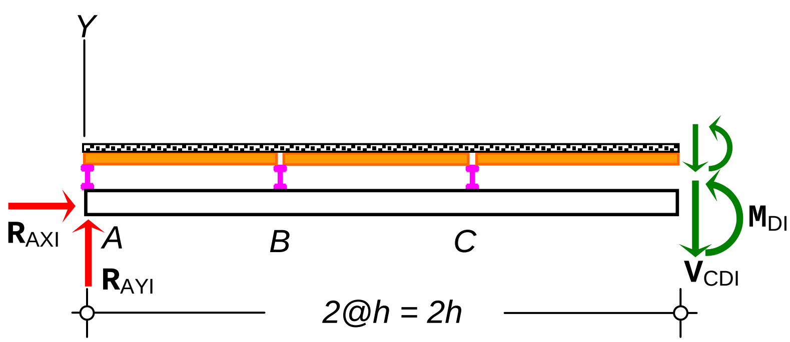
$$\mathbf{V}_{CDI} = \mathbf{V}_{CD}^{ext} = \mathbf{R}_{AYI} - \mathbf{L}_{left}$$
$$\mathbf{M}_{CI} = \mathbf{M}_{C}^{ext} = \mathbf{R}_{AYI}(2h) - \mathbf{M}_{C,left} \quad (17)$$

where $\mathbf{V}_{CDI}$ *and* $\mathbf{M}_{CI}$ *are influence lines of the shear force in a panel CD and the bending moment at a point C, respectively;* $\mathbf{L}_{left}$ *is the sum of load transmitted to all panel points appearing in the FBD;* $\mathbf{M}_{C,left}$ *is the sums of moment due to load transmitted to all panel points appearing in the FBD about points C.*

By introducing a fictitious section within the panel CD at point just to the left of point D, the corresponding free body diagram is obtained below.



Unit load is on the left of the section



Unit load is on the right of the section

Free body diagram

Equilibrium of this portion of the floor system dictates that

$$\mathbf{V}_{CDI} = \mathbf{V}_{CD}^{ext} = \mathbf{R}_{AYI} - \mathbf{L}_{left}$$
$$\mathbf{M}_{DI} = \mathbf{M}_{D}^{ext} = \mathbf{R}_{AYI}(3h) - \mathbf{M}_{D,left} \qquad (18)$$

where $\mathbf{V}_{CDI}$ *and* $\mathbf{M}_{DI}$ *are influence lines of the shear force in a panel CD and the bending moment at a point D, respectively;* $\mathbf{L}_{left}$ *is the sum of load transmitted to all panel points appearing in the FBD;* $\mathbf{M}_{D,left}$ *is the sums of moment due to load transmitted to all panel points appearing in the FBD about points D.*

By comparing the equilibrium equations for the truss (16), (14), (15) and the equilibrium equations for the equivalent floor system (17) and (18) and using the relation (13) and fact that the load transferring systems of both structures are equivalent, i.e. $\mathbf{L}_{\text{left}} = L_{\text{left}}$, $\mathbf{M}_{\text{C,left}} = M_{\text{C,left}}$ *and* $\mathbf{M}_{\text{D,left}} = M_{\text{D,left}}$, *we then obtain*

$$\begin{aligned} V_{\text{CD}} &\equiv -F_{\text{CJI}} \sin\theta = \mathbf{V}_{\text{CDI}} \\ M_{\text{C}} &\equiv -F_{\text{IJI}} \cdot a = \mathbf{M}_{\text{CI}} \\ M_{\text{J}} &\equiv F_{\text{CDI}} \cdot a = \mathbf{M}_{\text{DI}} \end{aligned} \tag{19}$$

The analogy (19) concludes that

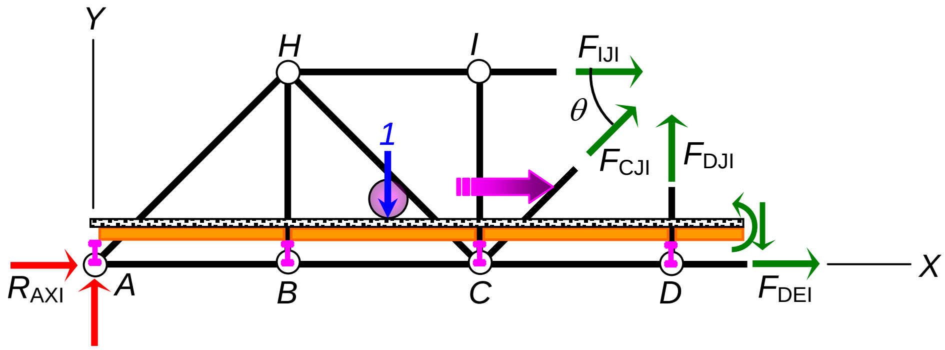
- *The resultant internal shear force of the loading panel of the truss through which the fictitious section passes is identical to the influence line of the shear force of the same panel of the equivalent floor system*
- *The resultant internal moment about a point C of the truss is identical to the influence line of the bending moment at a point C of the equivalent floor system*
- *The resultant internal moment about a point J of the truss is identical to the influence line of the bending moment at a point D of the equivalent floor system where the point D is the projection of the point J onto the loading path.*

Since the resultants internal shear force and the resultant internal moments are related to the influence lines of the internal forces of the truss members by (14), Equation (19) then yields the direct relations between the influence lines of the internal forces of the truss members and the influence lines of the shear force and the bending moment of the equivalent floor system. These relations are useful in the construction of the influence lines of the internal forces of the truss members since it is only required to obtain the expressions of the resultant internal shear forces and resultant internal moments in terms of the internal member forces of interest (14) and to construct the influence lines for the shear force and the bending moments of the floor system. The former task can be achieved by considering a FBD of a part of the truss resulting from the sectioning and the latter task can be readily accomplished by using Müller Breslau principle as discussed previously.

By following a similar procedure, the influence line of the internal member force F_{DJI} *can be readily obtained as follow. By introducing a fictitious section passing through the members IJ, CJ, DJ, and DE and considering the FBD of the left part of the truss as indicated in the figure below, we first obtain the resultant internal shear force in terms of the internal member forces appearing in the FBD and then form the following analogy*

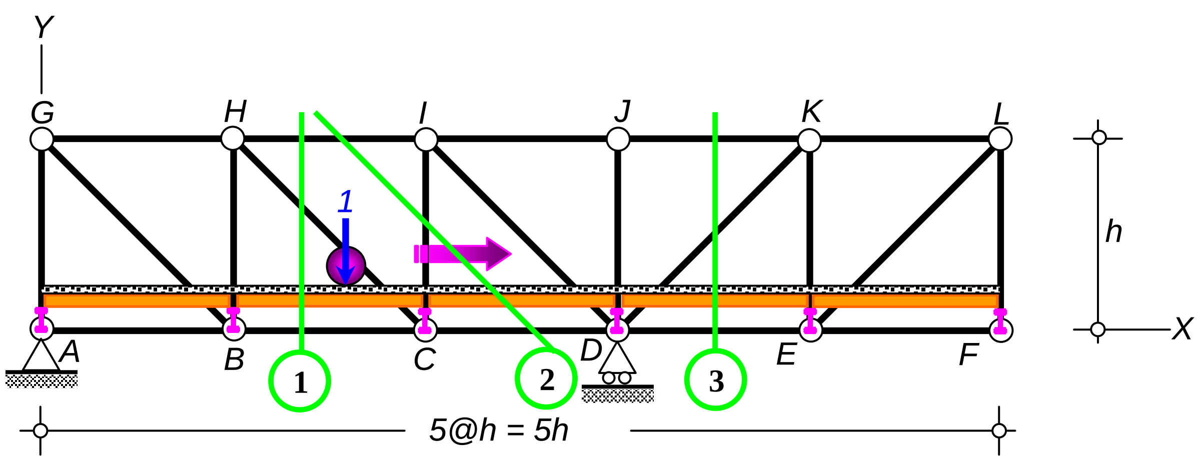
$$V_{\text{DE}} \equiv -F_{\text{CJI}} \sin\theta - F_{\text{DJI}} = \mathbf{V}_{\text{DEI}} \tag{20}$$

where $\mathbf{V}_{DEI}$ *is the influence line of the shear force in a panel DE of the floor system. Since the influence line* F_{DJI} *is already obtained from (19) and the influence line* $\mathbf{V}_{DEI}$ *can be readily obtained using Müller Breslau principle, (20) can be used to construct the influence line* F_{DJI}.



Free body diagram

Example12 Construct influence lines of the support reactions R_{AYI}, R_{DYI}, *and the internal member forces* F_{BCI}, F_{CHI}, F_{HII}, F_{CII}, and F_{DKI} of a statically determinate truss given below.



Solution *To use the truss-floor system analogy, we first establish an equivalent floor system for the given truss. This system must consist of five loading panels of total span length 5h and the pinned support and the roller support are located at the panel point A and the panel point D, respectively. The schematic of the equivalent floor system is depicted below*